

## LISTING OF CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the instant application.

1. (Currently amended) A method for controlling hourglass deformations of an under-integrated solid element in a finite element analysis for designing and analyzing a structural product, the method comprising:

establishing a local initial element coordinate system of the under-integrated solid element ~~for~~ from an initial undeformed geometry of the solid element;

establishing a local current element coordinate system of the solid element ~~for~~ from a current deformed geometry of the solid element;

calculating a set of initial nodal coordinates of the solid element in the local initial element coordinate system;

calculating a set of current nodal coordinates of the solid element in the local current element coordinate system;

evaluating a set of hourglass shape vectors of the solid element from the initial nodal coordinates;

calculating a set of hourglass deformation magnitudes of the solid element ~~using the set of hourglass shape vectors, and difference between the initial nodal coordinates and the current nodal coordinates of corner nodes of the solid element~~ as set forth in Equation as follows:

$$\text{Equation: } \hat{g}_{i\alpha} = \sum_{J=1}^N \bar{\gamma}_{\alpha J} (\hat{x}_{iJ} - \bar{X}_{iJ})$$

where:

N = total number of the corner nodes in the solid element,

J = one of the corner nodes,

$\hat{g}_{i\alpha}$  = one of the hourglass deformation magnitudes (spatial direction i for hourglass mode  $\alpha$ ).

$\bar{\gamma}_{\alpha J}$  = one of the hourglass shape vectors (node J for hourglass mode  $\alpha$ ).

$\bar{X}_{iJ}$  = one of the initial nodal coordinates (node J in spatial direction i of the local initial element coordinate system).

$\hat{x}_{iJ}$  = one of the current nodal coordinates (node J in spatial direction i of the local current element coordinate system);

evaluating a set of generalized hourglass forces from the hourglass deformation magnitudes, the local initial nodal coordinates, and material constants of the solid element; and

calculating a set of counter nodal forces for controlling the hourglass deformations in the local current element coordinate system from the generalized hourglass forces and the hourglass shape vectors, wherein the set of counter nodal forces is applied in directions opposing to ~~used to resist~~ the hourglass deformations such that the hourglass deformations are controlled in the finite element analysis ~~of~~ for designing and analyzing a the structural product.

2. (Previously presented) The method as recited in claim 1, further comprising:

transforming the set of counter nodal forces from the local current element coordinate system to global coordinate system before adding to global force array.

3. (Original) The method as recited in claim 1, further comprising:

calculating all terms of an element stabilization matrix for the solid element from the hourglass shape vectors, the initial nodal coordinates, and material constants of the solid element.

4. (Previously presented) The method as recited in claim 3, further comprising:

transforming the stabilization matrix from the local initial element coordinate system to global coordinate system before adding the terms of the stabilization matrix into global stiffness matrix.

5. (Original) The method as recited in claim 1, wherein the solid element is chosen from the group consisting of three-dimensional 8-node hexahedral element, 6-node three-dimensional pentahedral element, two-dimensional 4-node plane strain element and two-dimensional 4-node axisymmetric continuum element.

6. (Currently amended) A software product embodied in a tangible computer readable storage medium and executing in a computing device for controlling hourglass deformations of an under-integrated solid element in a finite element analysis for designing and analyzing a structural product, the software product comprising:

program code for establishing a local initial element coordinate system of the under-integrated solid element ~~for from~~ an initial undeformed geometry of the solid element;

program code for establishing a local current element coordinate system of the solid element ~~for from~~ a current deformed geometry of the solid element;

program code for calculating a set of initial nodal coordinates of the solid element in the local initial element coordinate system;

program code for calculating a set of current nodal coordinates of the solid element in the local current element coordinate system;

program code for evaluating a set of hourglass shape vectors of the solid element from the initial nodal coordinates;

program code for calculating a set of hourglass deformation magnitudes of the solid element as set forth in Equation as follows:

Equation: 
$$\hat{g}_{i\alpha} = \sum_{J=1}^N \bar{y}_{\alpha J} (\hat{x}_{iJ} - \bar{X}_{iJ})$$

where:

N = total number of the corner nodes in the solid element,

J = one of the corner nodes,

$\hat{g}_{i\alpha}$  = one of the hourglass deformation magnitudes (spatial direction i for hourglass mode  $\alpha$ ),

$\bar{y}_{\alpha J}$  = one of the hourglass shape vectors (node J for hourglass mode  $\alpha$ ),

$\bar{X}_{iJ}$  = one of the initial nodal coordinates (node J in spatial direction i of the local initial element coordinate system),

$\hat{x}_{iJ}$  = one of the current nodal coordinates (node J in spatial direction i of the local current element coordinate system) using the set of hourglass shape vectors, and difference between the initial nodal coordinates and the current nodal coordinates of corner nodes of the solid element;

program code for evaluating a set of generalized hourglass forces from the hourglass deformation magnitudes, the initial nodal coordinates, and material constants of the solid element; and

program code for calculating a set of counter nodal forces for controlling the hourglass deformations in the local current element coordinate system from the generalized hourglass forces and the hourglass shape vectors, wherein the set of counter nodal forces is applied in directions opposing to used to resist the hourglass deformations such that the hourglass deformations are controlled in the finite element analysis of for designing and analyzing a the structural product.

7. (Currently amended) The software product as recited in claim 6, further comprising:

program code for transforming the set of counter nodal forces from the local current element coordinate system to global coordinate system before adding to global force array.

8. (Original) The software product as recited in claim 6, further comprising:

program code for calculating all terms of an element stabilization matrix for the solid element from the hourglass shape vectors, the initial nodal coordinates, and material constants of the solid element.

9. (Previously presented) The software product as recited in claim 8, further comprising:

program code for transforming the stabilization matrix from the local initial element coordinate system to global coordinate system before adding the terms of the stabilization matrix into global stiffness matrix.

10. (Original) The software product as recited in claim 6, wherein the solid element is chosen from the group consisting of three-dimensional 8-node hexahedral element, 6-node three-dimensional pentahedral element, two-dimensional 4-node plane strain element and two-dimensional 4-node axisymmetric continuum element.

11. (Currently amended) A system for controlling hourglass deformations of an under-integrated solid element in a finite element analysis for designing and analyzing a structural product, the system comprising:

an I/O interface;

a data communications interface;

a memory for storing computer readable code for an application module;

at least one processor coupled to the memory, the I/O device and the data communications interface, said at least one processor executing the computer readable code in the memory to cause the application module to perform operations of:

establishing a local initial element coordinate system of the under-integrated solid element ~~for~~ from an initial undeformed geometry of the solid element;

establishing a local current element coordinate system of the solid element ~~for~~ from a current deformed geometry of the solid element;

calculating a set of initial nodal coordinates of the solid element in the local initial element coordinate system;

calculating a set of current nodal coordinates of the solid element in the local current element coordinate system;

evaluating a set of hourglass shape vectors of the solid element from the initial nodal coordinates;

calculating a set of hourglass deformation magnitudes of the solid element as set forth in Equation as follows:

$$\text{Equation: } \underline{\hat{g}_{i\alpha}} = \sum_{J=1}^N \bar{\gamma}_{\alpha J} (\hat{x}_{iJ} - \bar{X}_{iJ})$$

where:

N = total number of the corner nodes in the solid element,

J = one of the corner nodes,

$\hat{g}_{i\alpha}$  = one of the hourglass deformation magnitudes (spatial direction i for hourglass mode  $\alpha$ ),

$\bar{\gamma}_{\alpha J}$  = one of the hourglass shape vectors (node J for hourglass mode  $\alpha$ ),

$\bar{X}_{iJ}$  = one of the initial nodal coordinates (node J in spatial direction i of the local initial element coordinate system),

$\hat{x}_{iJ}$  = one of the current nodal coordinates (node J in spatial direction i of the local current element coordinate system) using the set of hourglass shape vectors, and difference between the initial nodal coordinates and the current nodal coordinates of corner nodes of the solid element;

evaluating a set of generalized hourglass forces from the hourglass deformation magnitudes, the initial nodal coordinates, and material constants of the solid element; and

calculating a set of counter nodal forces for controlling the hourglass deformations in the local current element coordinate system from the generalized hourglass forces and the hourglass shape vectors, wherein the set of counter nodal forces is applied in directions opposing to ~~used to resist~~ the hourglass deformations such that the hourglass deformations are controlled in the finite element analysis ~~of~~ for designing and analyzing a the structural product.

12. (Previously presented) The system of claim 11, further comprising operations of:

transforming the set of counter nodal forces from the local current element coordinate system to global coordinate system before adding to global force array.

13. (Previously presented) The system of claim 11, further comprising operations of:

calculating all terms of an element stabilization matrix for the solid element from the hourglass shape vectors, the initial nodal coordinates, and material constants of the solid element.

14. (Previously presented) The system of claim 13, further comprising operations of:

transforming the stabilization matrix from the local initial element coordinate system to global coordinate system before adding the terms of the stabilization matrix into global stiffness matrix.

15. (Previously presented) The system of claim 11, wherein the solid element is chosen from the group consisting of three-dimensional 8-node hexahedral element, 6-node three-dimensional pentahedral element, two-dimensional 4-node plane strain element and two-dimensional 4-node axisymmetric continuum element.